

# Teaching Authentic Problem Solving

DEVELOPING GREATER MATHEMATICAL REASONING WITH STUDENTS



## Do Now:

What are the challenges students have when asked to complete application problems in your classroom?

---

---

---

---

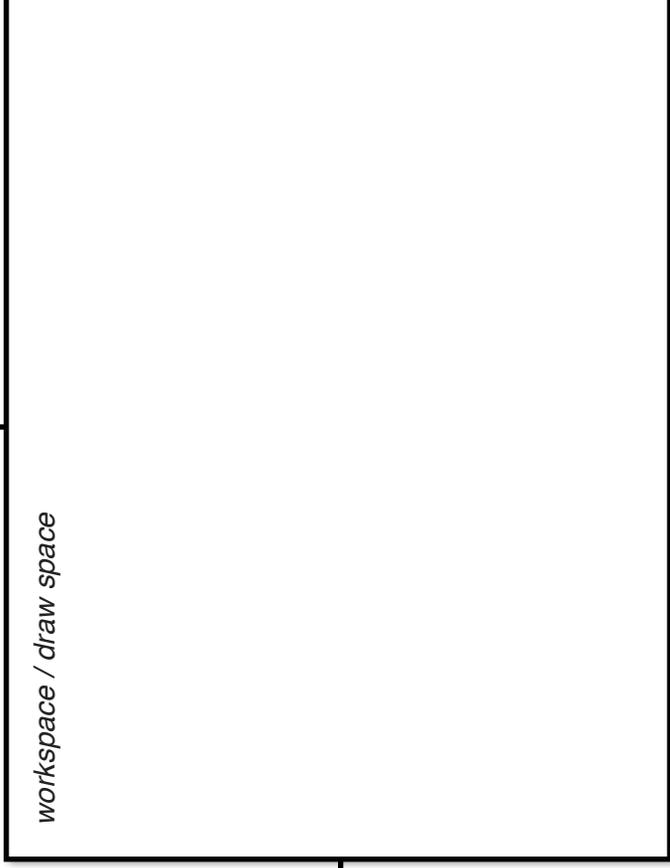
---

---

---

*What is the problem asking?*

*How would I figure this out? (my method)*

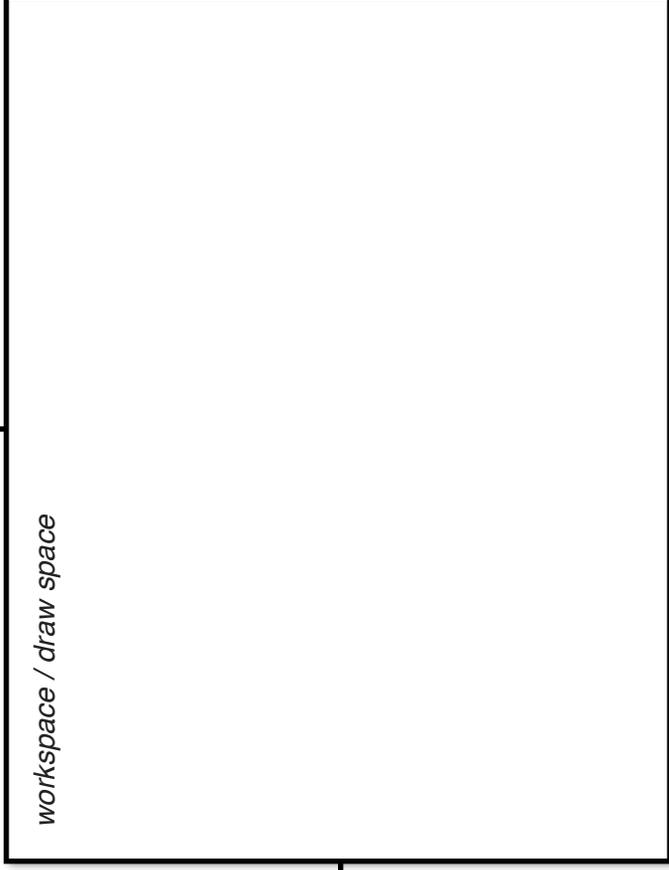


*What information do I need to know?*

*How can I get the information I don't have?*

*What is the problem asking?*

*How would I figure this out? (my method)*



*workspace / draw space*

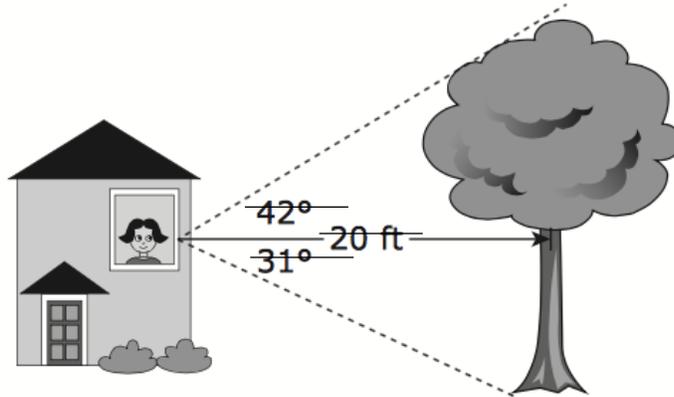
*What information do I need to know?*

*How can I get the information I don't have?*

Planning guide for:

	Student Misconceptions	Guiding questions to <b>support</b> students
<p><i>How would I figure this out? (method/steps)</i></p> <p>-</p> <p>-</p> <p>-</p> <p>-</p>	<p>-</p> <p>-</p> <p>-</p> <p>-</p> <p>-</p>	
<p><i>What information do I need to know?</i></p>		
<p><i>How can I get the information I don't have?</i></p>		

Mariela is standing in a building and looking out of a window at a tree. The tree is 20 feet away from Mariela. Mariela's line of sight to the top of the tree creates a  $42^\circ$  angle of elevation, and her line of sight to the base of the tree creates a  $31^\circ$  angle of depression.



Present students with just the question, and allow them opportunities to determine *how* they can go about solving the problem, and what information they would need to do so.

The focus is on the process, *not* on the answer.

What is the height, in feet, of the tree?

**Teacher Dialogue**

*“What is the question asking?”*

*“Go ahead and answer the question”*

*“How could you conclusively determine the height of the tree? What could you do?”*

*“What information would you like to have?”*

**Purpose**

Clarification to ensure students know what is being asked of them.

Allows students to try to answer the question and determine on their own that they need more information.

Prompts students to develop and justify their **process**.

Requires students to determine what they need and provide a rationale for why it is needed.

**Benefits of presenting problems where focus is on student reasoning:**

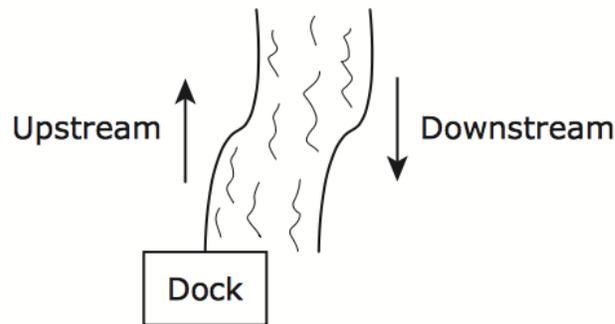
- Generates mathematical discussion among students.
- Students themselves determine *what is needed* (pieces of information) to answer the question, and *how* (steps and mathematical operations needed) to answer the question.
- The teacher *guides* students but does not explicitly show them how the problem is done (productive struggle).
- Students have opportunity to employ (and thus become more comfortable with) different problem-solving strategies.

When learning word problems, students are often

- presented with all information before knowing the question to be answered
- told in advance the formula needed to solve a formula-based word problem
- asked to identify what information is necessary *before* having determined the **process** needed to answer the question.
- given a set of steps to follow and an almost identical example of what they will later be asked to do independently.
- rarely given opportunities to dialogue about the process and the math involved

All of these common practices hinder students' development in important reasoning and critical thinking skills necessary for problem-solving.

Gabriel operates a riverboat and frequently offers tours of the river. Typically, a tour lasts for 3.25 hours. The riverboat usually takes 2.00 hours to make the 25-mile trip upstream from the dock and 1.25 hours to make the 25-mile return trip downstream.



Gabriel is considering offering a shorter tour that will last 2.50 hours and travel only 20 miles upstream before returning. Will the shorter tour be possible if the riverboat travels at the same speed as it does in the 3.25-hour tour? Show your steps and justify your answer.

**When faced with an application problem, the thought process of a student should be:**

**With authentic problem solving practice, students will develop stronger mathematical reasoning.**

*What is the question asking me to do?*

example:

*I need to figure out if Gabriel has enough time to get the boat upstream and back downstream in 2.5 hours using a shorter route.*

*How would I figure this out?*

*I need to determine how long it takes the boat to go upstream and back downstream.*

*What information do I need to know?*

*The speed of the boat going each way, and the distance to where the boat turns around.*

*How can I get the information I need?*

*I can figure out speed with the distance given and the time it takes for the boat to complete each part of the longer tour. Then with that I can determine how long each part would take for the shorter tour.*